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An economic analysis of input structure in context to information inaccuracy, improvement and predictions

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Abstract

During the last six decades, the information theory has attracted the researchers from worldwide and its literature is growing leaps and bounds. Some of its terminologies even have become part of our daily language. Every probability distribution has some uncertainty associated with it. The concept of 'entropy' is introduced here to provide a quantitative measure of this uncertainty. Different approaches for measure of entropy and its development has been made, viz: 1.An axiomatic approach, 2.Measure of entropy via measure of inaccuracy and directed divergence and 3.Information measures and coding theorem. A hypothetical data of agricultural, fisheries and forestry sectors, in each of nine years were framed. All inputs bought to fisheries and forestry sectors were supplied by other firms of the same sector. It was worked out that the smaller the distance of probability distribution P from Q, the greater will be the uncertainty and greater the entropy. This is always positive and vanishes if and only if P = Q. Now from the Shannon entropy

$$D(P/Q) = \sum p_i \log \frac{p_i}{1/n} = \log n - \left(-\sum p_i \log p_i\right)$$
$$= \log n - H(P)$$

The calculation of D.F. Kerridge inaccuracy same as we did for measures using the Kullback – Liebler measure of relative information. So that as probability becomes more and more probability equal as the probability distributions comes closer to the uniform distribution, D(P/Q) becomes smaller and smaller, H(P) decreases larger and larger till H(P) approaches log n as P approaches Q.

Keywords: Entropy, probability, inaccuracy, predictions, coding

Every probability distribution has some uncertainty associated with it. The concept of 'entropy' is introduced here to provide a quantitative measure of this uncertainty. Entropy is a measure of disorder or randomness of a system with a large number of constituents and assumes its maximal value when a system can be in a number of states randomly with equal probability and is minimally zero when the system is in a specific state, with no uncertainty in its description.

During the last six decades, the information theory has attracted the researchers from worldwide and its literature is growing leaps and bounds. Some of its terminologies even have become part of our daily language. The subject has now developed to such an extent that it is being applied in many quite different disciplines such as biology, chemistry, cybernetics, economics, linguistics, statistical inference and statistical ecology, computer sciences, pattern recognition, fuzzy sets etc.

We restricted ourselves only to those aspects of information theory which are closely related to our work. Different approaches for measure of entropy and its development has been made, viz:

1. An axiomatic approach.
2. Measure of entropy via measure of inaccuracy and directed divergence.
3. Information measures and coding theorem.

Measure of entropy via measure of inaccuracy and directed divergence:

Suppose an experimenter asserts that the probabilities of the events $E_1, \ldots, E_K$ are $Q = (q_1, \ldots, q_K) \in \delta_K$ while the true probabilities are $P = (p_1, \ldots, p_K) \in \delta_K$. The asserted probabilities may naturally be different from the true ones on the following two counts.

(i) The available information may not be enough and hence the statement may be vogue and

(ii) Because of wrong information the statement may be incorrect.

Kerridge (1961) has given a concept of inaccuracy which takes into account both these aspects . For P, Q, $\in \delta_K$ the measure of this inaccuracy is formulated via

$$H_K(P;Q) = -\sum_{i=1}^{K} p_i \log q_i \tag{1.2.1}$$

Here the assumptions that whenever any q_i is zero, corresponding p_i also is zero and the convention 0 log 0 = 0 is adopted. The expression (1.2.1) can be written as

$$H_K(P;Q) = -\sum_{i=1}^{K} p_i \log p_i + \sum_{i=1}^{K} p_i \log(p_i/q_i)$$
$$= H_K(P) + E_K(P;Q) \tag{1.2.2}$$

Kerridge called E_K (P; Q) "inaccuracy of error". Obviously when $p_i = q_i$ for each i, E_K (P; Q) = 0 and then the Kerridge inaccuracy is nothing but the entropy due to Shannon. Thus the Kerridge inaccuracy is a generalization of Shannon's entropy. Kerridge himself gave an axiomatic characterization of H_K(P; Q).

By employing the idea of inaccuracy measures, the relation between noiseless coding and entropies have further been enhanced. Suppose that two persons A and B believe that probability of i^{th} event is q_i and that the code words with lengths $n_1, n_2, \ldots, n_K$ has been constructed accordingly. But contrary to their belief the true probability is p_i . Such a code is called 'personal probability code' and a result is established as

$$L \geq H(P,Q) \geq -\sum_{i=1}^{K} p_i \log_D q_i \tag{1.2.3}$$

This coding theory approach if n_i to be an integer just greater than equal to $-\log_D p_i$, we get the minimum codeword length which corresponds to a measure of entropy. However, we take another set of codeword lengths given by n_i integer just greater than or equal to $-\log_D q_i$, where $q_1, q_2, \ldots, q_K$ is another probability distribution. We get a mean codeword length lying between $-\sum p_i \log q_i$ and $-\sum p_i \log q_i + 1$. The measure of inaccuracy defined by eqn. (1.2.1) is expressed as the measure of inaccuracy is greater than equal to the measure of entropy and the two measures coincides only if we use the minimizing codeword lengths.

In this approach we obtain entropy of a probability distribution in terms of its 'distance' from the most uncertain distribution i.e. U. The smaller the distance of probability distribution P from U, the greater will be the uncertainty and the greater will be the entropy. The first measure of directed divergence of a probability distribution P from probability distribution Q was given by Kullback and Liebler (1951) defined as

$$D(P,Q) = \sum_{i=1}^{k} p_i \log \frac{p_i}{q_i} \tag{1.2.4}$$

This is always ≥ 0 and vanishes iff p = Q. Now from Shannon's entropy

$$D(P,U) = \sum_{i=1}^{n} p_i \log \frac{p_i}{1/n} =$$

$$\log n - \left(\sum_{i=1}^{n} p_i \log p_i\right) = \log n - H(P) \tag{1.2.5}$$

So that as the probabilities become more and more equal i.e. as the probability distribution comes closer to the uniform distribution, D(P,U) becomes smaller an smaller, H(P) becomes larger and larger till H(P) approaches log n as P approaches U.

The study was undertaken with the following objectives:

1. To propose the improved information measures on univariate and bivariate distributions and their relationships in coding with respect to conventional measures.
2. To examine the measures of information empirically.

Methodology

Suppose in the light of an experiment (we mean, after taking observations) the set of probabilities are revised from q_i to p_i (i.e. 1, …, k). Then, Kullback and Leibler (1951) relative information provided by an experiment is defined via

$$I_m(P;Q) = \sum_{i=1}^{K} p_i \log(p_i/q_i) \tag{3.4.1}$$

This measure has found applications in statistical inference and estimation. It is assumed that whenever any q_i is zero, then corresponding p_i is also zero and we take $0\log_2\left(\frac{0}{0}\right) = 0$. Kullback (1959) called it "divergence", Kerridge "error" Renyi (1960) and Aczel (1968) "information gain".

Detailed study of this measure, with its applications in statistics would be found in Kullback. Theil (1967) has given its several applications in economic analysis. In this context an important issue in input-output analysis is to what extent the input structure of the various sectors is stable over time. The procedure is applicable for measuring the inaccuracy of various types. The inaccuracy of the forecast of the input structure of agriculture, forestry and fishery of second year (the 2nd column of the table 1 over the first year (the first column of the table 1) have been calculated by utilizing the equation (1). Next, third year is to be predicted on the basis of the Ist year data, and, so on, the information inaccuracy of the forecast is measured (which are not consecutive) which gives the complete information of all input structure forecasts for the agriculture, forestry and fisheries in the years 1st through year 9th. The first row contains the information inaccuracy values which are obtained when the first year input structure is used to forecast the structure of second year, third year, ….. up to ninth year. The second row uses the second year data, used to predict all later years, third, fourth, so on up to ninth and so on. The figures increase from left to right in each row.

In the same way, the forecast two years ahead, three years ahead and so on are calculated. The results are shown in Table 2 which contains the average information values of the agriculture, forestry and fisheries sectors. For large span of time value, the aggregation procedure is applied.

Averaging procedure is applicable to calculate the diagonal figures of the table 2 that are all inaccuracy values corresponding to forecasts one year ahead. In the same way forecasts two years ahead, three year ahead, and, so on, may be calculated. This, we call average inaccuracy. The results are shown in table 3, which contains the average inaccuracy values of agriculture, forestry and fishery. This procedure is applicable for measuring the various types of inaccuracies of the forecast of the input structure of agriculture forestry and fishery.

Results and Discussion

A hypothetical data of agricultural, fisheries and forestry sectors, in each of nine years were framed. All inputs bought to fisheries and forestry sectors were supplied by other firms of the same sector. This includes the seed, feed plants by one from other. Furthermore, some percentage were supplied by food manufacturing sector, chemical sector, chemical and petroleum refineries, services rendered by wholesaler, some percentage is the share of all other sectors which are their supplier to agriculture, fisheries and forestry, goods and services supplied by economic agent outside the domestic enterprises system. Another is wage paid to hired labours. There is also depreciation on fixed assets such as loan interest, and net profit, etc.

This input – output system is such that total inputs equal to total output for each sector, necessity to be exhaustive with respect to the various inputs and it included depreciation and net profit.

Table 1 shows the input structure of a country in nine years containing agriculture, forestry and fisheries, which has been supplied by various sectors of economy. The sectors are other firms of agriculture, fisheries and forestry, food manufacturing, chemical and petroleum refineries sector, wholesale trading sectors, all other sectors, import sectors, wages sectors and gross profit sectors. As per profitability norms these total supplied sectors are accounted as one. In the first year other firms of agriculture, fisheries and forestry supplied by 18.60%, food manufacturing sectors 8.00%, chemical and petro refineries 3.50%, wholesale traders 1.50%, all other sectors 9.10%, imports 4.70%, wages.

14.20% and gross profit 40.40% respectively supplied to the main sector. Similarly in the next years these sectors supplied to the main sector and so on. A close perusal of the table indicates the decreasing trend of input supply in years to year ahead. This shows the probability is in decreasing orders implies that the uncertainty increases and create information more. In this scenario it is important to mention that in case of reduction in probability entropy increases and getting more information to reduce the uncertainty.

(1) Information inaccuracy of the forecast of agriculture, forestry and fishery on the basis of second year over the first year

$$= \left(0.175 \log_e \frac{0.175}{0.186}\right) + ... + \left(0.422 \log_e \frac{0.422}{0.404}\right) * 1.443 \text{ bits}$$

$$= 0.00365 \text{ bits}$$

(2) Suppose the 3rd year is to be predicted on the basis of first year data, the information inaccuracy is then

$$= 1.443 * \left(0.167 \log_e \frac{0.167}{0.186} + ... + 0.453 \log_e \frac{0.453}{0.404}\right)$$

$$= 0.01519 \text{ bits}$$

The second row uses the second year data, to predict all later years i.e. third year, fourth year, … , eighth year. We observe that figures of the Table 2 increases row wise systematically when we move from left to right in each row.

The number of information inaccuracy values obtained is large and even larger when we carry out the same procedure for the input structure. The number of bits will very from year to year. Since the information concept is essentially additive. A natural measure for inaccuracy of a number decomposition forecasts is the average information inaccuracy given in Table 3. We notice that the diagonal figures of the Table 2 (365, 544, 129, … , 50 all in 10^{-5} bits) are inaccuracy values corresponding to forecasts one year ahead. In the same way, the forecasts two year ahead, with inaccuracy values (1519, 783, …., 32 in 10^{-5} bits and so on).

Table 1. Structure of inputs supplied by different sectors of an economy towards Agriculture, Fisheries and Forestry sector over different years (Hypothetical)

S. No.	Particulars	1st year	2nd year	3rd year	4th year	5th year	6th year	7th year	8th year	9th year
1	Other firms of Agri., Fisheries and Forestry	0.186	0.175	0.167	0.163	0.161	0.159	0.156	0.153	0.15
2	Food manufacturing sector	0.080	0.089	0.098	0.090	0.088	0.085	0.083	0.082	0.083
3	Chemical and Petroleum Refineries sector	0.035	0.037	0.033	0.035	0.033	0.031	0.030	0.030	0.032
4	Whole sale traders sector	0.015	0.015	0.015	0.017	0.015	0.016	0.017	0.015	0.017
5	All other sectors	0.091	0.094	0.083	0.081	0.079	0.077	0.075	0.073	0.073
6	Imports sector	0.047	0.042	0.041	0.04	0.038	0.035	0.033	0.035	0.032
7	Wages sector	0.142	0.126	0.111	0.107	0.105	0.103	0.101	0.102	0.103
8	Gross profit	0.404	0.422	0.453	0.467	0.481	0.494	0.505	0.510	0.501
9	Total Input	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 2. Information of eight years of the input structure of Agriculture, Forestry and Fisheries

Base year (t)	01	02	03	04	05	06	07	08
00	365	1519	1836	2333	2970	3639	3779	3892
01		534	783	1164	1690	2245	2402	2439
02			129	261	563	915	1004	1067
03				74	249	485	569	582
04					71	231	260	310
05						46	84	106
06							39	32
07								50

Note: All information values expressed in 10^{-5} bits.

Table 3. Average Information Inaccuracy for different time spans of predication

Time span (t)	Agri., Fisheries and Forestry averaging procedure	Averaging improvement by taking the successive difference of the elements of column
(1)	(2)	(3)
I year	163	
II year	450	450 – 163 = 287
III year	736	736 – 450 = 286
IV year	1163	1163 – 736 = 427
V year	1700	1700 – 1163 = 1584
VI year	2369	2369 – 1700 = 669
7^{th} to 8^{th} year	3370	3370 – 2369 = 1001

Note: All information value expressed in 10^{-5} bits.

The results are shown in table 3 which contains the average information values of the agriculture sector. This agrees as regards the increase with an increasing time span t. For large τ values some aggregation is applied. Since the number of observations would be rather small. The prediction year 7 to 8 implies that the average inaccuracy value is taken for all cases for which τ = 7, τ = 8 and similarly for 7 - 10.

The averaging procedure is applicable. The diagonal figures of the table are all inaccuracy value corresponding time t. Now we consider this result in the light of the prediction revisions. Our aim is to predict input structure of some sector in year 7. Suppose that first year is the most recent year for which input data are available, so that τ = 7 years. Suppose that one year later we have secondary data and that we are still interested in 7 yrs. Then we are in a position to make a prediction revision. The original information inaccuracy forecast is

$$D(P/Q) = \sum p_i \log p_i/q_i \qquad (4.4.1)$$

where $q_1, q_2, \ldots, q_K$ represent the I yr input structure and $p_1, p_2, \ldots, p_K$ that of 7 years (the year to be predicted). One year later, we have the 2^{nd} year input structure at our disposal. This is the revised forecast of the P_i to be indicated by $q_1', q_2', \ldots, q_K'$. Their information inaccuracy is

$$D(P/Q') = \sum p_i \log p_i/q_i' \qquad (4.4.2)$$

Our expectation that the new inaccuracy is less than its predecessor at least on average otherwise the revision makes matter worse rather than better. So we subtract the new value from the old one.

$$D(P/Q) - D(P/Q') = \sum p_i \log \frac{q_i'}{q_i} \qquad (4.4.3)$$

This is the information improvement of the forecast revisions q_i'. It is positive when there is indeed an improvement in the sense that the information inaccuracy is reduced, zero when the inaccuracy remains unchanged and negative when the inaccuracy is larger than it was before the revision. One can get average improvement from table 3 by taking the successive difference of the element of each column.

Now we calculate the Shannon Entropy of table 1

I Entropy of the 1st column of the table 1 is

$$H(P)_{I\,r} = \left(0.186 \log_e \frac{1}{0.186}\right) + \left(0.08 \log_e \frac{1}{0.08}\right) +$$

$$\ldots + \left(0.404 \log_e \frac{1}{0.404}\right) * 1.443 \text{ bits}$$

II Entropy of the second column of the table 4.4.1 is

$$= \left(0.175 \log_e \frac{1}{0.175}\right) + \ldots$$

$$+ \left(0.422 \log_e \frac{1}{0.422}\right) * 1.443 \text{ bits}$$

= 2.427 bits

We proceed as usual, we find the entropy of the different columns of years to years, given in table 4.

(iii) Entropy of the third column, fourth column, …., ninth column given as (Table 4):

We observe that the number of entropy values obtained in table 4 is considerably smaller as could be expected. It seems that entropy is small and it becomes even smaller year to ahead with span of time t. Since information is always a measure of decrease of entropy. This implies that the gaining of information by reducing our doubts or chaos. In this way we are in a sound position to get the information year wise about the input structure of agriculture, forestry and fisheries sectors. Therefore we concluded that the probability decreases, entropy (information) increase. Since information is proportional to entropy, so information is improved which satisfies the relation given in (4).

Table 4. Entropy for different span of time

Column	Entropies
I	2.449 bits
II	2.427 bits
III	2.363 bits
IV	2.342 bits
V	2.229 bits
VI	2.265 bits
VII	2.238 bits
VIII	2.225 bits
IX	2.232 bits

Table 5. D.F. Kerridge inaccuracy

Base year (t)	D.F. Kerridge Inaccuracy							
	01	02	03	04	05	06	07	08
00	2.452	2.464	2.468	2.472	2.479	2.486	2.489	2.488
01		2.483	2.435	2.439	2.445	2.45	2.452	2.452
01			2.365	2.366	2.369	2.369	2.372	2.373
03				2.342	2.344	2.346	2.347	2.348
04					2.30	2.302	2.302	2.303
05						2.266	2.660	2.660
06							2.239	2.238
07								2.2385

The result of Kerridge inaccuracy given in table (5), minus the Shannon entropy of table (4) is the direct result of table (2), which satisfies the relation given in (2). This is the inaccuracy of errors or vice versa. Since the relative measure is a measure of distance (directed divergence) between two probability distributions in terms of its distance from the most uncertain distribution Q.

The smaller the distance of probability distribution P from Q, the greater will be the uncertainty and greater the entropy. This is always positive and vanishes if and only if P = Q. Now from the Shannon entropy

$$D(P/Q) = \sum p_i \log \frac{p_i}{1/n} = \log n - \left(-\sum p_i \log p_i\right)$$
$$= \log n - H(P)$$

The calculation of D.F. Kerridge inaccuracy same as we did for measures using the Kullback – Liebler measure of relative information. So that as probability becomes more and more probability equal as the probability distributions comes closer to the uniform distribution, D(P/Q) becomes smaller and smaller, H(P) decreases larger and larger till H(P) approaches log n as P approaches Q.

Summary

The concept of Shannon entropy (1948) is the central role of information theory referred as measure of uncertainly. The entropy of a random variable is defined in terms of its probability distribution and can be shown to be a good measure of randomness of uncertainties. This chapter mainly deals with its different approaches for the measure of entropy, information content, requirements of measure of uncertainty of probability distribution and their axiomatic approach and properties for discrete finite random variable are studied. The study is extended to measure of entropy via inaccuracy and directed divergence for two probability distributions. The third approach was initiated by Shannon himself in which he introduced the concept of information-theoretic entropy.

Comprehensive review of literature is the most important for any research work. The chapter second gave an explanatory and specific relevant literature to understand the objectives of the present study. It mainly contained two major sub-headings viz.

- Shannon additive entropy and its generalizations, and
- Non-additive entropies.

The chapter third deals with an understanding the mathematical foundation and its applications to the chapter 4, for the development of the information measures and their relation to coding theory was developed. The following preliminary is given under the following sub-heads.

1. Necessity of generalization of measure of information and inaccuracy.
2. Hölder's inequality
3. Useful directed divergence measure
4. Kullback – Leobler's relative information measure and their procedure for calculations.

In presence of the objectives of the present study on "Some improved measures of information" were discussed in detailed study in various aspects have been examined. The results were obtained and proved the coding theorems for univariate and bivariate probability distributions. The chapter four is further divided into four sections of the following sub-headings viz.

1. Noiseless coding theorems for univariate probability distributions
2. Noiseless coding theorems on personal probability codes for bivariate probability distribution.
3. Useful directed divergence and its applications.
4. The information inaccuracy of input structure prediction and its information improvement.

Noiseless coding theorems for univariate probability were tried to express for two parametric generalization of order α of the power distribution P^{β}, and established the upper bounds and lower bounds in terms of average code length under the given conditions. It is mentioned to note that

(i) When all code word length are equal, each of these means reduces to size n.

(ii) Each mean lies between min $(n_1, n_2, \ldots, n_k)$ and max $(n_1, n_2, \ldots, n_k)$.

(iii) If code word lengths $(n_1, n_2, \ldots, n_k)$ are increased by α, then each mean is increase by α.

Noiseless coding theorems on personal probability codes for two distributions have been discussed by employing the idea of inaccuracy. The relations between noiseless coding and entropies have been further strengthened. We have proved coding theorems for personal probability codes by considering generalized measure of inaccuracy for incomplete probability distributions associated with the generalized average code lengths under the given constraints have been established. Some properties of $L_R^{\beta}(P)$ are also discussed.

Further, we have discussed the properties, utility information function and useful directed divergence for two parameters with its application. We introduced a function using utility information associated the average code length with utility aspect and established a result that in a way, give the characterization of the utility information measure satisfying the condition. We also obtained the bounds for all integers D > 1 as

$$L_{\alpha}\left(P^{\beta},N;U\right) > H_{\alpha}\left(P^{\beta}/Q;U\right).$$

According as R $\gtrless$ 1.

In next section, from different tables, input – output structure in span of time were discussed. A close perusal of the tables indicated the trends of the data, which give a large of information to reduce the uncertainty.

References

Aczel, J. 1968. On different characterizations of entropies: Probability information theory. Proc. Internat. Symp., McMaster Univ. Flamilton, Ontario.

Acze1, J. and Daroczy, Z. 1975. *On measures of information and their characterizations*. Academic Press, New York.

Arimoto, S. 1971. Information - theoretical considerations on estimation problems. *Information and Control* **19**:181- 194.

Autar, R. and Soni, R.S. 1975. Inaccuracy and coding theorem. *J. Appl. Prob* **12**:845-851.

Autar, R. and Soni, R.S. 1976. Generalized inaccuracy and a coding theorem. Sec (A), *Proc. Indian Acad. Sci* **84**(5) **:**204-209.

Beckenhack, F.F. and Bellman, R. 1971. *Inequalities*. Third Printing. Springer - Verlag, New York.

Behara. M. and Nath, P. 1973. Additive and non-additive entropies of finite measurable partitions: Probability and Information Theory II. *Lecture Notes in Math.* **296**:102-138. Berlin - Heidelberg - New York.

Belis, M. and Guiasu, S. 1968. *Quantitative - qualitative measure of information in cybernetic systems*, pp 11-14, 593-594. IEEE Trans. Information Theory.

Chaundy, T.W. and Mcleod. J.B. 1960. On a functional equation. *Proc. Edinburgh Math. Soc. Notes*, **43**: 7-8.

Deepa, R. 1980. On axiomatic characterization of second order information improvement. *Jour. Comb. Infor. Syst. Sci* **5**:107-111.

Feinstein, A. 1958. *Foundations of Information Theory*. Mc Graw-Hill, New York.

Guiasu, S. 1977. *Information Theory with Applications*. McGraw-Hill, New York.

Gupta, H.C. and Sharma, B.D. 1975. On non-additive measure of inaccuracy. *Czech. Math. J.* **26**(101): 584-594.

Hartley, R.V.L. 1928. Transformation of information. *Bell Svst. Tech. Jour.* **7**: 535-563.

Havdra, J.F. and Charvat, F. 1967. Quantification method of classification process: The concept of structural α - entropy. *Kybernetika* **3**: 30-35.

Hobson, A. 1969. A new theorem of information theory. *J. Statist. Phys.* **1**: 383-391.

Hooda, D.S. and Kumar 2004. Generalized measures of useful directed divergence and information improvement with Applications. *Defense Science Journal* **54**(02): 125-133.

I. Vajda 1970. On the amount of information contained in a sequence of independent observations. *Kybernetika* **6**(4): 306-324.

Ilyas, M. and Saud, I 1984. Entropy and internal information of Printed Urdu. *Jour. Appi. Statist.* **11**(2): 142-154.

Kannappan P.L. 1972. On Shannon's entropy, directed divergence and inaccuracy. *Z. Wahr. Verw. Geb.* **22**: 95-100.

Kannappan, P.L. and Rathie, P.N. 1972. Application of a functional equation to information theory. *Ann. Polon. Math* **26**: 95-101.

Kannappan, P.L. 1974. On generalization of some measures in information theory. *Glas Mat. Ser.* III **9**(29): 81-93.

Kapur, J.N. 1967. Generalized entropy of order α and type β. *The Mathematical Seminar* **4**: 78-94.

Kendall, D.G. 1964. Functional equations information theory, Z. Wahr. Verw. Geb. **2**: 225-229.

Kerridge, D.F. 1961. Inaccuracy and inference. *Jour. Royal Statist. Soc. Ser. B* **23**: 184-194.

Khan. A.B. and Zaheeruddin 1977. On generalized functional equations in information theory. *Proc. ISI Bulletin* **47**(4): 276-279.

Khan, A.B. 1979. On useful information of order α of the power distribution P^{β}. Proceedings of 14th annual conference on Statistics, Computer Science and Operation Research. Institute of Statistical Studies and Research, Egypt, **2**: 89-95.

Khan, A.B., Bhat, B.A. and Pirzada, S. 2005. Some results on a generalized useful information measure. *J. inequalities in Pure and Applied Mathematics* **6**(4): 117.

Kullback, S and Lieibler, R.A. 1951. On information and sufficiency. *Annual Math Statistics* **22**: 79-86.

Kullback, S. 1959. *Information Theory and Statistics*. John Wiley and Sons, New York.

Longo, G. 1976: A noiseless coding theorem for sources having utilities. *SIAMJ. Applied Mathematics* **30**(4): 739-748.

Losonczi, L. 1981. A characterization of entropies of degree α. *Metrika*, **28**:237-244.

Mathai, A.M. and Rathie, P.N. 1975. *Basic Concept in Information Theory and Statistics*. Wiley Eastern Ltd., New Delhi.

Nyquist, H. 1924 Certain factors affecting telegraph speed. *Be11 Syst. Tech. Jour.* 324.

Rathie, P.N. 1967. Generalized entropies in coding theory. Internat. Conf. on system sciences. Honolulu, Jan. 1968. (*Metrika*, **10**: 216-219, 1972).

Rathie, P.N. 1970. On a generalized entropy and a coding theorem. *Jour. Appi. Prob.* **7**: 124-133.

Rathie, P.N. 1970a. On generalized measures of inaccuracies, information and errors in information. *Statistica* **30**: 340-349.

Rathie, P.N. 1972. Generalized entropies in coding theory. *Metrika* **18**: 216-219.

Rathie, P.N. and Kannappan, P.L. 1972. A directed-divergence function of type β. *Inform and Control* **20**: 38-45.

Rathie, P.N. and Kannappan. P.L. 1973. An inaccuracy function of type β. *Ann. Inst. Statist. Math* **25**: 205-214.

Renyi. A 1960. Some fundamental problems of information theory. *Magar Tud. Akad. Mat. Fiz. Oszt. Kozl* **10:**251-282.

Renyi, A 1961. *On measures of entropy and information*, pp. 547-561. Proc. Fourth Brekeley Symp. Math. Statist. Ad Prob., 1960, University of California Press **1**: 547-561.

Roy, L.K. 1976. Comparison of Renyi entropies of power distribution. *ZAMM* **56**: 217-218.

Saud, I. and Khan, M.Z. 1984. On generalized R-norm information measures. *Gujrat Statist. Review* **11**(2): 55-68.

Saud, I. and Khan, M.Z. 1984. Characterization of R-norm information measures. *Jour. Statist. Research* **18:** 45-55.

Saud, 1. and Khan, M.Z. 1986. Axiomatic Characterization of information improvement of type. *Jour. Statist. Research*, **20**: 23-39.

Shannon, C.E. 1948. A mathematical theory of communication. *Bell. System Tech. J* **27**:379-423, 623-656.

Sharma, B.D. and Autar, R. 1947. Information improvement functions. *Econometrica* **42**: 103-112.

Sharma, B. D. 1970. *The mean value study of quantities in information theory*. Ph.D. thesis, University of Delhi.

Sharma, B. D. and Autar, R. 1974. Relative information functions and their type (α,β) generalization. *Metrika* **21**: 41-50.

Sharma, B. D. and Taneja, I. J. 1974. On axiomatic characterization of information-theoretic measures. *J. Statist. Phys.* **10**(4): 337-346.

Sharma, B.D. and Mittal 1975. New Non-additive measures of entropy for discrete probability distribution. *Journal of Mathematical Science* **10**: 28-40.

Taneja, I. J. 1976. On axiomatic characterization information improvement and its generalization, *Journal of Combinatorics, Information and System Science* **1**(2): 69-79, New Delhi, India.

Taneja, I.J. 1976a. On information Improvement and its generalization. *Metrika* Band **23**: 249-257.

Taneja. I.J. 1976b. On measures of information and inaccuracy. *J. Statist. Physc* **14**: 203-270.

Taneja. I.J. 1977. On a Characterization of unified measure of information theory. *Inform. Sd* **13**:229-237.

Taneja. I.J. 1979. Some Contributions to Information Theory-I: On Measures of Information (A Survey). *J. Comb. Inform, and Syst. Sci.* **4**: 253-274.

Taneja. T.J. 1984. Lower Bounds on Exponentiated Mean Codeword Length for the Best 1:1 Code. *Matemática Aplicada e Compzit.* **3**: 199-204.

Taneja. I.J. 1984a. On Characterization of Generalized Information Measures. *J. Comb. Inform, and Sysi. Sd.* **9**: 169-174.

Taneja. I.J. 1987. Statistical aspects of divergence measures. *J. Stat. Plann. and Inferen.* **16**: 137-145.

Taneja. 1.J. 1988. Bivariate Measures of Type and Their Applications. *Tamkang J. Math.* **19**(3): 63-74.

Taneja. I.J. 1989. *On Generalized Information Measures and Their Applications*, pp327-413. Chapter in:Ad. Elect. and Elect. Physics, Ed. P.W. Hawkes.

Taneja. I.J. 1990. *On Generalized Entropies with Applications*, pp 107-169. Chapter in: Lectures in App!. Math. and Inform., Ed. L.M. Ricciardi, Manchester University Press.

Taneja. T.J. 1990a. Bounds on the Probability of Error in Terms of Generalized Information Radius. *Inform. Sd.*, bf **51**

Taneja, I.J. 1995. *New Developments in Generalized Information Measures*, pp 37-135. Chapter in: Advances in Imaging and Electron Physics, Ed. P.W. Hawkes.

Theil, H. 1967. *Economics and Information theory*. North Holland Publishing Co. Amsterdam.

Tukey, J.W., Ian C.R., Verna B. 1973. *Index to statistics and probability*. R and D Press.