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Modeling Long Memory in Volatility for Spot Price of Lentil with Multi-step Ahead Out-of-sample Forecast Using AR-FIGARCH Model

Ranjit Kumar Paul, Bishal Gurung, Sandipan Samanta and A.K. Paul

ICAR-Indian Agricultural Statistics Research Institute, New Delhi, India
Corresponding author: ranjitstat@gmail.com

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Abstract

The potential presence of long memory (LM) properties in return and volatility of the spot price of lentil in Indore market has been investigated. Geweke and Porter-Hudak (1983) (GPH) method has been applied to test for presence of long range dependence in the volatility processes for the series. Stationarity of the series has been tested using Augmented Dickey-Fuller (ADF) unit root test and Philips-Peron (PP) unit root test. It is observed that both the log returns as well as squared log returns series are stationary at level but there is a significant presence of long memory in squared log return series. Accordingly, AR-FIGARCH model was applied for forecasting the volatility of lentil price. To this end, window based evaluation of forecasting is carried out with the help of Mean squares prediction error (MSPE), Root MSPE (RMSPE), Mean absolute prediction error (MAPE) and Relative MAPE (RMAPE). The residuals of the fitted models were used for diagnostic checking. Out-of sample forecast of volatility has been computed for 1st June-31st July, 2015 along with the percentage deviation from the actual price. The maximum deviation has been found to be 2.55%. The R software package has been used for data analysis.

Keywords: Conditional heteroscedastic, lentil price, return series, stationarity, validation

There has been a large amount of research on long memory in economic and financial time series. The presence of long memory in asset returns has important implications for many of the models used in modern financial economics. Long run persistence or long memory in stock return volatility has important implications for predicting future volatility. For modelling the time series in presence of long memory, the autoregressive fractionally integrated moving-average (ARFIMA) model is used.

ARFIMA model uses a fractional parameter, *d*, to difference the data to capture long memory. Regarding long memory, very good descriptions can be found in Robinson (1995) and Baillie *et al.* (1996), who considered the developments in the econometric modelling of long memory, and Beran (1995) reviews long-memory modelling in other areas. The existence of non-zero *d* is an indication of long memory and its departure from zero measures the strength of long memory. Paul (2014) and Paul *et al.* (2015a,b) have applied ARFIMA model for forecasting of agricultural commodity prices.

However, ARFIMA model is unable to capture the volatility if present in the dataset. Many financial time series shows periods of stability followed by unstable periods with high volatility. To take care of the volatility, Engle (1982) proposed autoregressive conditional heteroscedastic (ARCH) model. But, ARCH model has the property that the unconditional autocorrelation function of squared residuals; if it exists; decay very rapidly compared to what is typically observed unless maximum lag is large. To overcome the weaknesses of ARCH model, Bollerslev (1986) and Taylor (1986) proposed the Generalized ARCH (GARCH) model independently of each other. Huge amount of empirical and theoretical research work has been already done for GARCH and related models.

Fung *et al.* (1994) described that a long memory process could allow conditional heteroscedasticity, which could be the explanation of non-periodic cycles. It seems a long memory model is more flexible than an ARCH model in terms of capturing irregular behaviour. In the regard, Baillie *et al.* (1996) developed Fractionally

Integrated GARCH (FIGARCH) model. FIGARCH model is capable of explaining and representing the observed temporal dependencies of the financial market volatility in a much better way than other types of GARCH models. Jin and Frechette (2004) applied FIGARCH model for describing fourteen agricultural future price series. Bordignon *et al.* (2004) have introduced a FIGARCH model with seasonality, which allows for both periodic patterns and long memory behaviour in the conditional variance. It can also merge these two aspects allowing the model to be both periodic and having long memory components. Paul *et al.* (2015c) applied ARFIMA-FIGARCH model for modelling and forecasting of volatility with long memory in agricultural commodities prices in India. For estimation of parameters of FIGARCH model, the value of fractional differencing parameter *d* is estimated first and then this is used to obtain the estimation of other parameters (Lopes and Mendes, 2006; H¨ardle and Mungo, 2008). In the present investigation, an attempt has been made to apply FIGARCH model for modelling and forecasting of long memory time series of volatile log return price of lentil in Indore Market.

Lentil is one of the most important rabi crops in the country. India is the second largest producer of the lentil in the world after Canada. Indian production of this crop is around 10 lakh metric tons per year which is cultivated on about 14 lakh hectares of land. The crop is grown in the winter season in the states of Uttar Pradesh, Madhya Pradesh, Bihar, West Bengal, Rajasthan, Haryana, Punjab, Assam and Maharashtra. Around 90% of the production comes from the top four states of the country. The sentiments of traders play a significant role in governing the price of this crop. Prices are also influenced by imports and exports in other countries as well as substitution with other pluses such as chana, tur, yellow peas etc. India exports around one lakh tons of lentil every year. The country also imports nearly 50 thousand tons every year. The main destinations of exports are Sri Lanka, Egypt, UAE, Sudan, Yemen and Bangladesh. Imports are mainly from Canada, USA, Turkey and Australia.

In the present investigation, the volatility in the log returns series of spot price of lentil in Indore market is modeled using FIGARCH model. The paper is organized as follows: section 2 deals with the concept of long memory process along with testing the presence of long memory. Section 3 deals with testing presence of ARCH effect; section 4 describes FIGARCH model, its estimation and forecasting; results and discussion is given in section 5 followed by conclusion in section 6.

Long Memory Process

Long memory in time-series can be defined as autocorrelation at long lags Robinson (2003). According to Jin and Frechette (2004), memory means that observations are not independent (each observation is affected by the events that preceded it). The acf of a time-series y_t is defined as,

$$\rho_k = \text{cov}\ y_t, y_{t-1})/\ \text{var}\ y_t$$

for integer lag *k*. A covariance stationary time-series process is expected to have autocorrelations such that, $\lim_{k\to\infty}\rho_k = 0$. Most of the well-known class of stationary and invertible time-series processes have autocorrelations that decay at the relatively fast exponential rate, so that $\rho_k \approx |m|^k$, where $|m|<1$ and this property is true, for example, for the well-known stationary and invertible ARMA(*p*,*q*) process. For long memory processes, the autocorrelations decay at an hyperbolic rate which is consistent with $\rho_k \approx Ck^{2d-1}$, as *k* increases without limit, where C is a constant and *d* is the long memory parameter.

Long memory tests

Long memory is an important empirical feature of any financial variables. The presence of long memory in the data implies the existence of nonlinear forms of dependence between the first and the second moments, and thus the potential of time-series predictability. Testing for long memory property is an essential task since any evidence of long memory would support the use of Long Memory (LM)-based volatility models such as FIGARCH.

Long memory components in the returns series and squared returns series have been tested using the Geweke and Porter-Hudak (1983) (GPH) statistic. The test has been extensively used in the literature. For long memory in the volatility process, this test is applied to the logarithm of squared returns series of lentil, which is commonly regarded as a proxy of conditional volatility (Lobato and Savin, 1998; Choi and Hammoudeh, 2009).

Testing for ARCH effects

Let ε_t be the residual series. The squared series $\{\varepsilon_t^2\}$ is then used to check for conditional heteroscedasticity, which is also known as the ARCH effects. The test for conditional heteroscedasticity is the Lagrange Multiplier (LM) test due to Engle (1982), which is equivalent to usual F-statistic for testing H_0: $a_i = 0$, $i = 1, 2, \ldots, q$ in the linear regression,

$$\varepsilon_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + \ldots + a_q \varepsilon_{t-q}^2 + e_t,\ t = q+1, \ldots, T$$

where ε_t denotes error term, q is pre-specified positive integer and T is sample size. Let $SSR_0 = \sum_{t=q+1}^{T} \left(\varepsilon_t^2 - \bar{w}\right)^2$,

where $\bar{w} = \sum_{t=q+1}^{T} \varepsilon_t^2 / T$ is sample mean of $\left(\varepsilon_t^2\right)$ and

$SSR_1 = \sum_{t=q+1}^{T} \hat{e}_t^2$, where $\hat{e}_t$ is least squares residual of above regression model. Then, under H_0:

$$F = \frac{\left(SSR_0 - SSR_1\right)/q}{SSR_1\left(T - q - 1\right)}$$

is asymptotically distributed as chi-squared distribution with q degrees of freedom. The decision rule is to reject H_0 if $F > \chi_q^2(\alpha)$, where $\chi_q^2(\alpha)$ is the upper $100(1-\alpha)^{th}$ percentile of χ_q^2 or, alternatively, the p-value of F is less than α.

FIGARCH Process

Bollerslev (1986) and Taylor (1986) proposed the Generalized ARCH (GARCH) model independently of each other, in which conditional variance is also a linear function of its own lags and has the following form,

$$\varepsilon_t = \xi_t h_t^{1/2}$$

$$h_t = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} b_j h_{t-j}$$

$$= a_0 + a(L)\varepsilon_t^2 + b(L)h_t$$

where $\xi_t \sim \text{IID}(0,1)$. A sufficient condition for the conditional variance to be positive is,

The GARCH (p, q) process is weakly stationary if and only if $\sum_{i=1}^{q} a_i + \sum_{j=1}^{p} b_j < 1$.

The GARCH(p, q) process may also be expressed as an ARMA(m, p) process in ε_t^2,

$$\left[1 - a(L) - b(L)\right]\varepsilon_t^2 = a_0 + \left[1 - b(L)\right]v_t$$

Where, $m = \max\{p, q\}$ and $v_t = \varepsilon_t^2 - h_t$. The $\{v_t\}$ process can be interpreted as the "innovations" for the conditional variance, as it is a zero-mean martingale. Therefore, an integrated GARCH (p, q) process can be written as,

$$\left[1 - a(L) - b(L)\right](1 - L)\varepsilon_t^2 = a_0 + \left[1 - b(L)\right]v_t$$

The fractionally integrated GARCH or FIGARCH class of models is obtained by replacing the first difference operator $(1 - L)$ in above equation with the fractional differencing operator $(1 - L)^d$, where d is a fraction $0 < d < 1$. Thus, the FIGARCH class of models can be obtained by considering

$$\left[1 - a(L) - b(L)\right](1 - L)^d \varepsilon_t^2 = a_0 + \left[1 - b(L)\right]v_t$$

Such an approach can develop a more flexible class of processes for the conditional variance that are capable of explaining and representing the observed temporal dependencies of the financial market volatility in a much better way than other types of GARCH models (Davidson, 2004).

It may be noted that the fractional differencing operator $(1-L)^d$ can be written in terms of hypergeometric function,

$$(1 - L)^d = F(-d, 1, 1; L) = \sum_{k=0}^{\infty} \Gamma(k - d)\Gamma(k + 1)^{-1}\ \Gamma(-d)^{-1} L^k$$

The ARFIMA(p, d, q) class of models for the discrete time real-valued process $\{y_t\}$ introduced by Granger and Joyeux (1980); Granger (1980, 1981) and Hosking (1981) is defined by,

$$a(L)(1 - L)^d y_t = b(L)\xi_t$$

where $a(L)$ and $b(L)$ are polynomials in the lag operator of orders p and q respectively, and ξ_t is a mean-zero serially uncorrelated process. For the ARFIMA models, the fractional parameter d lies between $-1/2$ and $1/2$, (Hosking, 1981). The ARFIMA model is nothing but the fractionally integrated ARMA for the mean process. Analogous to the ARFIMA(*p, d, q*) process defined above for the mean, the FIGARCH(*p, d, q*) process for ε_t^2 can be defined as,

$$a(L)(1-L)^d \varepsilon_t^2 = a_0 + [1-b(L)]v_t$$

where $0 < d < 1$, and all the roots of $a(L)$ and $[1 - b(L)]$ lie outside the unit circle. In the case of ARFIMA model, the long memory operator is applied to unconditional mean μ of y_t which is constant. But this is not true in the case of FIGARCH model, where it is not applied to α_0, but on squared errors.

Estimation of FIGARCH Model

The estimation of parameters of FIGARCH model is generally carried out using the maximum likelihood method with normality assumption. But the normality assumption can be questioned with some empirical evidence and therefore the use of quasi-maximum likelihood estimator is preferred.

The FIGARCH model is estimated by using the quasi-maximum likelihood (QML) estimation method allowing for asymptotic normality distribution, based on the following log-likelihood function,

$$LL_T(\varepsilon_t,\theta) = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left[\log(h_t) + \frac{\varepsilon_t^2}{h_t}\right]$$

where $\theta' \equiv (a_0, d, b_1, b_2, \ldots, b_p, a_1, a_2, \ldots, a_q)$.

The likelihood function is maximized conditional on the start-up values. For the FIGARCH(*p, d, q*) model with $d > 0$, the population variance does not exist. In most practical applications with high frequency financial data, the standardized innovations $\xi_t = h_t^{-1/2}\varepsilon_t$ are leptokurtic and not normally distributed through time. In these situations the robust quasi-MLE (QMLE) procedures discussed by Weiss (1986) and Bollerslev and Wooldridge (1986) may give better results while doing inference. Baillie *et al.* (1996) have claimed the asymptotic normality of the quasi-maximum likelihood estimator, $\hat{\theta}_T$ when $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T)$ form a sample from FIGARCH(1*, d,* 0) by extending a similar result available for IGARCH(1,1), using a dominance-type argument. They have used an upper bound for the infinite sequence of coefficients of the ARCH(″) representation of an IGARCH model. A similar argument was also used in claiming the asymptotic properties of the quasi-maximum likelihood estimator for the FIGARCH. When estimating the parameters of a FIGARCH model, generally, the value of parameter *d* is estimated first and one uses these estimates to obtain the estimation of other parameters (Lopes and Mendes, 2006; H¨ardle and Mungo, 2008).

Forecasting by FIGARCH Model

Forecasting using a FIGARCH model has been discussed in (Tayafi and Ramanathan, 2012). The one-step ahead forecast of h_t is given by,

$$h_t(1) = a_0[1-b_1]^{-1} + \lambda_1\varepsilon_t^2 + \lambda_2\varepsilon_{t-1}^2 + \ldots$$

where, $\lambda_k \approx \left[(1-b_1)\Gamma(d)^{-1}\right]k^{d-1}$

Similarly, the two-step ahead forecast is given by,

$$h_t(2) = a_0[1-b_1]^{-1} + \lambda_1\varepsilon_{t+1}^2 + \lambda_2\varepsilon_t^2 + \ldots$$

Here ε_{t+1}^2 is unobservable and to be estimated by its conditional expectation $h_t(1)$, which is a function of past ε_t^2.

Therefore,

$$h_t(2) = a_0[1-b_1]^{-1} + \lambda_1 h_t(1) + \lambda_2\varepsilon_t^2 + \ldots$$

In general, the *l*-step ahead forecast is,

$$h_t(l) = \alpha_0[1-b_1]^{-1} + \lambda_1 h_t(l-1) + \ldots +$$
$$\lambda_l h_t(1) + \lambda_l\varepsilon_t^2 + \lambda_{l+1}\varepsilon_{t-1}^2 + \ldots$$

For all practical purpose, we stop at a large *M* and this leads to the forecasting equation,

$$h_t(l) \approx a_0[1-b_1]^{-1} + \sum_{i=1}^{l-1}\lambda_i h_t(l-i) + \sum_{j=0}^{M}\lambda_{l+j}\varepsilon_{t-j}^2$$

The parameters will have to be replaced by their corresponding estimates.

Results and Discussion

Daily time series data for spot prices of lentil in Indore Market during 1 January, 2007 to 31 July, 2015 has been considered. The return series are computed as differences in natural log prices. The data is collected form Ministry of Consumer's Affairs, Government of India. The data for the period January 1, 2007 to May 31, 2015 have been used for model building and the remaining data have been used for model validation. The summary statistics for return and squared return series have been computed and reported in table 1. A perusal of table 1 indicates that both series are positively skewed and platy-kurtic. The daily unconditional volatility of returns and the squared return, as measured by standard deviations, are 0.0127 and 0.0006 respectively.

The time series plot of spot price of lentil in Indore market, log return series and squared log return series have been exhibited in Fig. 1 to 3 respectively. A perusal of the figure 1 indicates that the spot price data is nonstationary; whereas the figure 2 and 3 depicts the stationarity pattern of log return and squared log returns series. In order to test for stationarity, two tests namely Augmented Dickey-Fuller (ADF) unit root test (Dickey and Fuller, 1979) and Philips-Peron (PP) unit root test (Philips and Perron, 1988) are used. The advantage of the PP tests over the ADF tests is that the PP tests are robust to general forms of heteroscedasticity in the error term and also the user does not have to specify a lag length for the test regression. In both the tests, the null hypothesis is "series is nonstationary" and alternative hypothesis is "the series is stationary".

The results of the tests are reported in Table 2. Table 2 indicates that both the returns as well as squared return series data are stationary but the spot price is nonstationary at level and stationary at first differenced data.

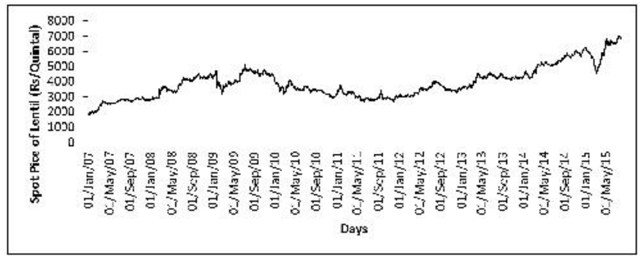

Fig. 1: Spot price of lentil at Indore

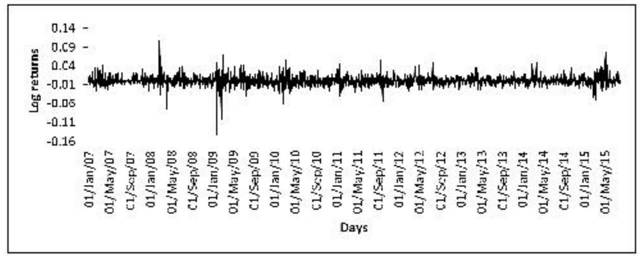

Fig. 2: Log returns series

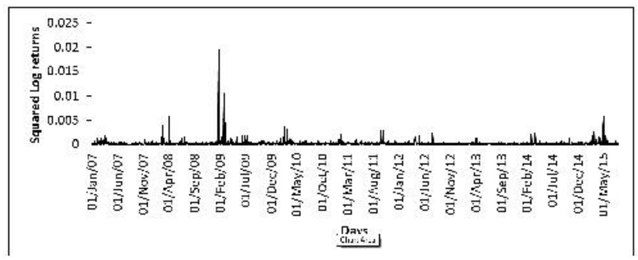

Fig. 3: Squared Log returns series

Presence of ARCH effect has been tested for both the log returns as well as squared log returns series. It is found that in squared returns series; there is significant presence of ARCH effect; whereas in the return series, there is no ARCH effect.

Table 1: Descriptive Statistics for spot price log Returns and squared log returns

Statistics	Spot price	Log return	Squared log return
Mean	3848.7450	0.0006	0.0002
Median	3650.0000	0.0000	0.0000
Maximum	7025.0000	0.1069	0.0196
Minimum	1817.5000	-0.1399	0.0000
Std. Development	983.8358	0.0127	0.0006
Skewness	0.8127	-0.3578	18.2276
Kurtosis	3.5709	16.9068	458.4408

Table 2. Test for stationarity

Series	ADF Test	PP Test
Spot price-level	-0.685	-0.649
Spot price-Ist difference	-20.681**	-39.022**
Return series	-20.515**	-38.318**
Squared return series	-17.026**	-38.105**

**denotes significant at 1% level

Autocorrelation

The distributional characteristics of the return series can be investigated further by analyzing the behavior of their autocorrelation functions. The results, displayed in Fig. 4 and 5, show that the autocorrelation functions of the returns are small and have no particular form. Most of them stay inside the 95% confidence intervals (the horizontal dotted line parallel to x-axis). This is suggestive of their short memory property. The autocorrelation functions of the squared returns are however larger, and they remain significant for many lags. More importantly, they exhibit a slow decay, indicating that the time series are strongly auto-correlated up to a long lag.

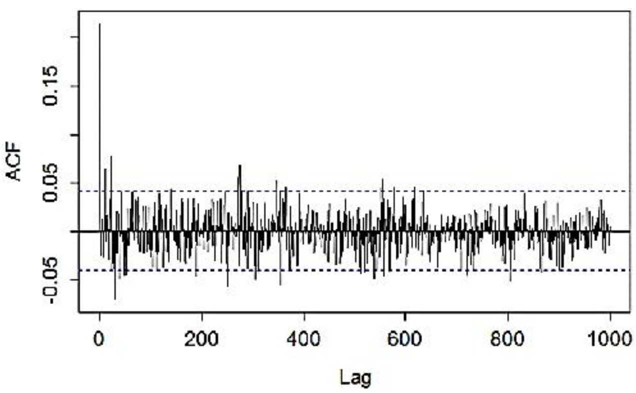

Fig. 4: Autocorrelation Function for log Returns

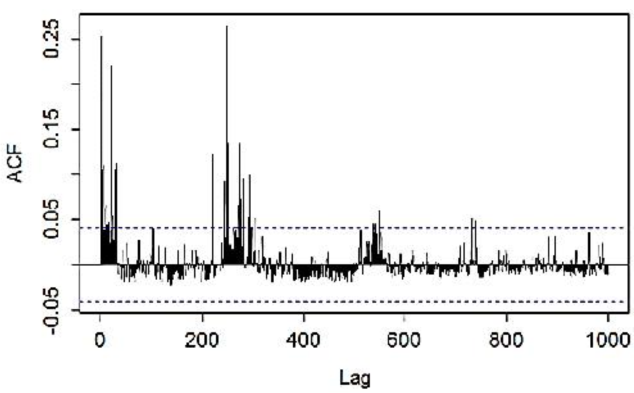

Fig. 5: Autocorrelation Function for squared log Returns

Results of long memory tests

We apply the GPH tests for testing long memory to the raw and squared returns of the spot prices of lentil. The obtained results are reported in Table 3. For the (raw) return series, the test shows no evidence of LM patterns; as the null hypothesis of no persistence is not rejected.

Table 3: Results of LM Tests for Returns and Squared Returns

Long memory parameter	Return	Squared return
d	-0.074	0.3497
SE	0.110	0.114
Z	0.061	3.182
P-value	0.951	<0.01

The result for squared return is different from that of the returns. Indeed, long memory property is found to be highly significant for the squared returns. Since squared returns are a good proxy for volatility, these findings thus suggest that the conditional volatility of return would tend to be range-dependent, persist and decay slowly. Intuitively, this volatility persistence can be appropriately modeled by a FIGARCH process because it allows for long memory behavior and slow decay of the impact of a volatility shock.

It is, however, important to note that the estimate of the LM parameter *d* is less than 0.5 for squared return indicating the stationarity of the process.

Fitting of FIGARCH Model

At first step, ARIMA model was fitted to the log returns series of lentil in Indore market. On the basis of autocorrelation function, partial autocorrelation function and AIC and BIC criteria, AR(1) models was found to be suitable for the data under consideration. In the next step, the residuals were checked for presence of ARCH effect. It is found that the autocorrelation functions are significant for the squared residual of fitted AR(1) model up to log lags and also ARCH – LM test is significant. So there is a significant presence of ARCH effect.

Table 4: Parameter estimate of AR(1)-FIGARCH(1,d,1) Model

Mean equation				
Constant	0.0002	0.0003	0.6391	0.5229
AR(1)	0.2464	0.0269	9.169	<0.001
Variance equation				
Constant	0.1119	0.0476	2.351	0.0188
d-Figarch	0.3966	0.0866	4.579	<0.001
ARCH(Phi1)	0.4174	0.1220	3.420	<0.001
GARCH(Beta1)	0.6000	0.1322	4.538	<0.001

On the basis of minimum AIC and BIC values, the best model identified for the data under consideration is AR(1)-FIGARCH (1,d,1) model. The parameters estimate of AR(1)-FIGARCH (1,d,1) model is reported in table 4. A perusal of table 4 indicates that, all the parameters are statistically significant. The long memory parameter, d is less than 0.5 ensures the stationarity of the model. The conditional variance of fitted FIGARCH model is reported in figure 6. It clearly indicates that conditional variance is very much time dependent.

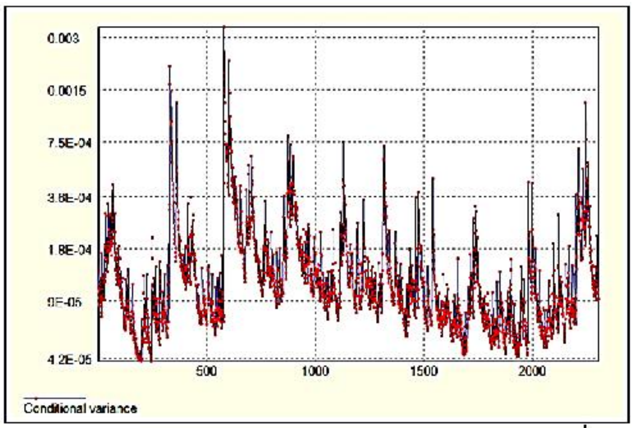

Fig. 6: Conditional variance of fitted FIGARCH model on the returns series

Diagnostic Checking

The model verification is concerned with checking the residuals of the model to see if they contained any systematic pattern which still could be removed to improve the chosen FIGARCH Model. This has been done through examining the autocorrelations and partial autocorrelations of the residuals of various lags. For this purpose, autocorrelations of the residuals were computed and it was found that none of these autocorrelations was significantly different from zero at any reasonable level. This proved that the selected FIGARCH model was an appropriate model for capturing the volatility present in the data under study.

Validation

One-step ahead forecasts of volatility for the period June 01, 2015 to July 31, 2015 (total 50 data points excluding market holidays) in respect of above fitted model are computed. The accuracy of model has been checked based on 9 windows namely 5-step, 10 step, 15-step, 20-step, 25-step, 30-step, 35-step, 40-step and 43-step ahead forecast. For measuring the accuracy in fitted time series model, Mean square prediction error (MSPE), Root mean square prediction error (RMSPE), Mean absolute error (MAE) and Relative mean absolute prediction error (RMAPE) are computed by using the formulae given below and are reported in Table 5.

$$\text{MAE} = 1/h\sum_{i=1}^{h}\left|y_{t+i} - \hat{y}_{t+i}\right|,$$

$$\text{MSPE} = 1/h\sum_{i=1}^{h}\left\{(y_{t+i} - \hat{y}_{t+i})^2\right\}$$

$$\text{RMSPE} = \left[1/h\sum_{i=1}^{h}\left\{(y_{t+i} - \hat{y}_{t+i})^2\right\}\right]^{1/2}$$

$$\text{RMAPE} = 1/h\sum_{i=1}^{h}\left\{\left|y_{t+i} - \hat{y}_{t+i}\right|/y_{t+i}\right\}\times 100,$$

where, h denotes the window length

A perusal of table 5 indicates that percentage error for all the window length is less than 1% representing a very good forecasting by the model. The actual and forecast value along with the percentage deviation is also reported in table 6 to verify the forecast individually. Here maximum deviation has been found to be 2.553 % on 17th July.

Table 5: Validation of Models

Window length	RMAPE(%)	MAE	MSPE	RMSPE
5	0.787	52.64	5762.56	75.91
10	0.600	40.02	3416.21	58.45
15	0.704	46.43	3669.81	60.58
20	0.616	40.58	3052.54	55.25
25	0.591	39.36	2978.07	54.57
30	0.677	37.12	2636.91	51.35
35	0.736	40.80	3310.87	57.54
40	0.729	39.00	3034.10	55.08
43	0.737	38.37	2899.63	53.85

The fitted model along with the actual data points is also depicted in Fig. 7 to visualize the performance of fitted models.

Conclusion

Several papers in the literature have addressed the issue of volatility modeling for commodity prices, but very few of them have actually investigated the nature and causes of the observed volatility persistence. The present investigation is aimed to fill this gap by testing the relevance of long memory in modeling the return

Table 6: Actual vs. predicted price (₹/Quintal)

Date	Actual Price	Predicted Price	Percent Deviation	Date	Actual Price	Predicted Price	Percent Deviation
01-Jun-15	6700	6544.940	2.314	01-Jul-15	6550	6558.008	0.122
02-Jun-15	6700	6744.203	0.660	02-Jul-15	6550	6552.080	0.032
03-Jun-15	6700	6702.128	0.032	03-Jul-15	6537.5	6552.080	0.223
04-Jun-15	6650	6702.128	0.784	06-Jul-15	6643.75	6536.623	1.612
05-Jun-15	6650	6640.340	0.145	07-Jul-15	6700	6671.243	0.429
08-Jun-15	6681.25	6652.112	0.436	08-Jul-15	6700	6715.503	0.231
09-Jun-15	6650	6690.785	0.613	09-Jul-15	6750	6702.128	0.709
10-Jun-15	6650	6644.742	0.079	10-Jul-15	6800	6764.025	0.529
11-Jun-15	6600	6652.112	0.790	13-Jul-15	6812.5	6814.041	0.023
12-Jun-15	6600	6590.325	0.147	14-Jul-15	6850	6817.624	0.473
15-Jun-15	6500	6602.096	1.571	15-Jul-15	6825	6861.076	0.529
17-Jun-15	6500	6478.634	0.329	17-Jul-15	7000	6821.268	2.553
18-Jun-15	6450	6502.065	0.807	20-Jul-15	7000	7044.269	0.632
19-Jun-15	6500	6440.278	0.919	21-Jul-15	7025	7002.223	0.324
22-Jun-15	6575	6513.948	0.929	22-Jul-15	7000	7033.158	0.474
23-Jun-15	6600	6594.955	0.076	23-Jul-15	7000	6996.323	0.053
24-Jun-15	6600	6608.024	0.122	24-Jul-15	6950	7002.223	0.751
25-Jun-15	6612.5	6602.096	0.157	27-Jul-15	6900	6940.433	0.586
26-Jun-15	6600	6617.560	0.266	28-Jul-15	6887.5	6890.418	0.042
29-Jun-15	6525	6599.143	1.136	29-Jul-15	6850	6886.734	0.536
30-Jun-15	6550	6509.458	0.619	30-Jul-15	6800	6843.336	0.637
				31-Jul-15	6800	6790.387	0.141

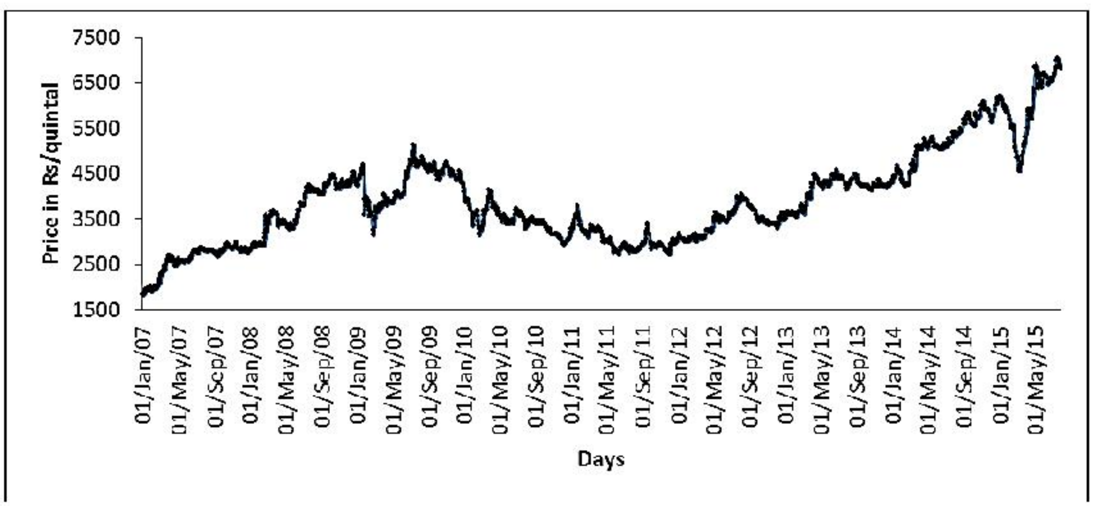

Fig. 7: Observed vs. predicted spot price of Lentil

and volatility for the spot prices of lentil. Application of GPH test indicated the existence of long memory in the volatility processes. On the basis of minimum AIC and BIC values, the AR(1)-FIGARCH(1, d, 1) model was found to be the best model for describing long memory in volatility. The sample ACFs of the volatility processes decay hyperbolically as the lag increases, indicating long-term memory exists in the squared log return series. In the context of the current financial situation, there is an increasing interest by traders, investors, portfolio managers, physical users and producers, and policy makers to understand better the performance and the distributional characteristics of increasingly important asset classes. Such enhanced understanding should lead to better returns, greater benefits from portfolio diversification, more adequate pricing of derivatives and improvement in risk management strategies. We find that long memory is particularly strong and plays a dominant role in explaining the spot price return of lentil. Finally, our out-of-sample analysis indicates that the FIGARCH-based model performs satisfactorily in terms of MASPE, MAPE and RMAPE.

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