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Some Arithmetic Operations on Triangular Intuitionistic Fuzzy Number and its Application in Solving Linear Programming Problem by Simplex Algorithm

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Abstract

The fuzzy logic and fuzzy numbers have been applied in many fields such as operation research, differential equations, fuzzy system reliability, control theory and management sciences etc. The fuzzy logic and fuzzy numbers are widely used in engineering applications also. In this paper we first describe Triangular Intuitionistic Fuzzy Number (TIFN) with arithmetic operations and solve a linear programming problem by Triangular Intuitionistic Fuzzy Number (TIFN) using simplex algorithm.

Keywords: Fuzzy set, Intuitionistic Fuzzy Sets, Intuitionistic Fuzzy Number, Triangular Intuitionistic Fuzzy Number (TIFN), Simplex algorithm

A fuzzy set in a universe X is defined by membership function that maps X to the interval [0, 1] and therefore implies a linear, i.e. total ordering of the elements of X, one could argue that this makes them inadequate to deal with incomparable information. A possible solution, however, was already implicit in Zadeh's[37] seminal paper in a footnote; he mentioned that "in a more general setting, the range of the membership function can be taken to be a suitable partially ordered set P." In every sector of our life, there arise several problems which can be formulated mathematically as optimization problem with the goal to maximize the profit or to minimize the cost to formulate the problem mathematically, some constrains or restrictions are to be considered. Linear programming is a one of the most important operation research technique and it is applied in many sector especially related to the optimization problem. Linear programming was first introduced by George Dantzig in 1947. Linear programming is a technique that is to optimize the use of limited resources. Formulation of fuzzy linear programming was first introduced by Zimmermann. Deldago[27] makes a general model of fuzzy linear programming within the limits of technical coefficients fuzzy and fuzzy right side. Fung and Hu[35] introduced the linear programming with the technique coefficients based on fuzzy numbers. Verdegay[27] define the dual problem through parametric linear program and shows that the problem of primal - dual fuzzy linear program has the same solution. In this paper we consider the linear programming problem in its standard form to find out it's feasible and optimal solution. We use simplex algorithm by trapezoidal intuitionistic fuzzy number to solve the linear programming problem.

Basic concept of Intuitionistic Fuzzy Sets

Atanassov (1983)[1] presented the concept of IFS, and pointed out that this single value combines the evidence for $x_i \in X$, but does not indicate evidence against $x_i \in X$. An IFS $\overset{\sim i}{A}$ in X is characterised by a membership function $\mu_{\overset{\sim i}{A}}(x)$ and a non -membership function $\nu_{\overset{\sim i}{A}}(x)$. Here, $\mu_{\overset{\sim i}{A}}(x)$ and $\nu_{\overset{\sim i}{A}}(x)$ are associated with each point in X, a real number in $[0,1]$ with the value of $\mu_{\overset{\sim i}{A}}(x)$ and $\nu_{\overset{\sim i}{A}}(x)$ at X representing the grade of membership and non-membership of x in $\overset{\sim i}{A}$. Thus, the closer the value of $\mu_{\overset{\sim i}{A}}(x)$ to unity and the value of $\nu_{\overset{\sim i}{A}}(x)$ to zero, the higher the grade of membership is, and the lower the grade of non-membership of x. When $\overset{\sim i}{A}$ is an ordinary set, its membership function (non-membership function) can take on only two values, 0 and 1. If $\mu_{\overset{\sim i}{A}}(x)=1$ and $\nu_{\overset{\sim i}{A}}(x)=0$, the element x does not belong to $\overset{\sim i}{A}$, similarly, if $\mu_{\overset{\sim i}{A}}(x)=0$ and $\nu_{\overset{\sim i}{A}}(x)=1$, the element x does not belong to $\overset{\sim i}{A}$. An IFS becomes a fuzzy set $\overset{\sim i}{A}$ when $\nu_{\overset{\sim i}{A}}(x)=0$, but $\mu_{\overset{\sim i}{A}}(x)\in[0,1]\forall x\in \overset{\sim i}{A}$.

Definition: Intuitionistic Fuzzy Set

Let a set X be fixed. An IFS $\overset{\sim i}{A}$ in X is an object having the for $\overset{\sim i}{A}=\left\{\left\langle x,\mu_{\overset{\sim i}{A}}0,1(x),\nu_{\overset{\sim i}{A}}(x)\right\rangle : x\in X\right\}$ where $\mu_{\overset{\sim i}{A}}(x): X\rightarrow[0,1]$ and $\nu_{\overset{\sim i}{A}}(x): X\rightarrow[0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x\in X$ to the set $\overset{\sim i}{A}$, which is a subset of X, for every element of $x\in X$, $0<\mu_{\overset{\sim i}{A}}(x)+\nu_{\overset{\sim i}{A}}(x)<1$.

Definition: (α,β)-cuts

A set of (α,β)-cuts, generated by IFS $\overset{\sim i}{A}$, where $\alpha,\beta\in[0,1]$ is a set of fixed numbers such that $\alpha+\beta\le 1$ is defined as

$$\overset{\sim i}{A}_{\alpha,\beta}=\begin{Bmatrix}\left(x,\mu_{\overset{\sim i}{A}}(x),\nu_{\overset{\sim i}{A}}(x)\right): x\in X\\ \mu_{\overset{\sim i}{A}}(x)\ge\alpha,\nu_{\overset{\sim i}{A}}(x)\le\beta,\alpha,\beta\in[0,1]\end{Bmatrix}$$

(α,β)-cuts denoted by $\overset{\sim i}{A}_{\alpha,\beta}$, is defined as the crisp set of elements x which belong to $\overset{\sim i}{A}$, at least to the degree α and which does belong $\overset{\sim i}{A}$ to the degree β.

Definition: Intuitionistic Fuzzy Number

An IFN $\overset{\sim i}{A}$ is

- an intuitionistic fuzzy sub-set of the real line
- normal, i.e., there is an $x_0\in\Re$ such that $\mu_{\overset{\sim i}{A}}(x_0)=1\left(\nu_{\overset{\sim i}{A}}(x_0)=0\right)$
- Convex for the membership function $\mu_{\overset{\sim i}{A}}(x)$ i.e. $\mu_{\overset{\sim i}{A}}(\lambda x_1+(1-\lambda)x_2)\ge\min\left(\mu_{\overset{\sim i}{A}}(x_1),\mu_{\overset{\sim i}{A}}(x_2)\right)\forall x_1,x_2\in\Re,\lambda\in[0,1]$
- concave for the non-membership function $\nu_{\overset{\sim i}{A}}(x)$ i.e. $\nu_{\overset{\sim i}{A}}(\lambda x_1+(1-\lambda)x_2)\le\max\left(\nu_{\overset{\sim i}{A}}(x_1),\nu_{\overset{\sim i}{A}}(x_2)\right)\forall x_1,x_2\in\Re,\lambda\in[0,1]$

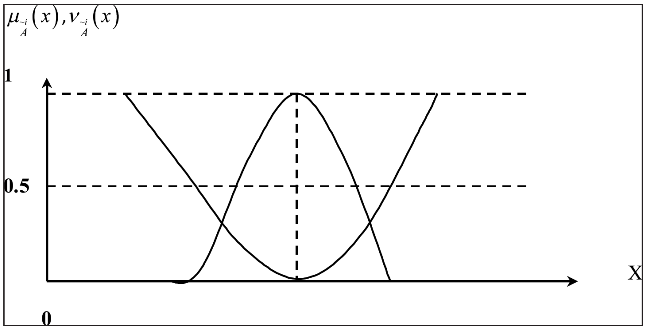

Fig. 1: Membership and non membership functions of $\overset{\sim i}{A}$

Definition: Triangular intuitionistic fuzzy number

A TIFN $\overset{\sim i}{A}$ is an IFN in R with the following membership function $\left(\mu_{\overset{\sim i}{A}}(x)\right)$ and non membership function $\left(\nu_{\overset{\sim i}{A}}(x)\right)$

$$\mu_{\overset{\sim i}{A}}(x)=\begin{cases} \dfrac{x-a_1}{b_1-a_1}, & a_1 \le x \le b_1 \\ \dfrac{c_1-x}{c_1-b_1}, & b_1 \le x \le c_1 \\ 0, & otherwise \end{cases} \quad \text{and}$$

$$\nu_{\overset{\sim i}{A}}(x)=\begin{cases} \dfrac{b_1-x}{b_1-a_1}, & a_1 \le x \le b_1 \\ \dfrac{x-b_1}{c_1-b_1}, & b_1 \le x \le c_1 \\ 1, & otherwise \end{cases}$$

Where $a_1^/ < a_1 < b_1 < c_1 < c_1^/$ and $\mu_{\overset{\sim i}{A}}(x), \nu_{\overset{\sim i}{A}}(x) \le 0.5$ for $\mu_{\overset{\sim i}{A}}(x)=\nu_{\overset{\sim i}{A}}(x) \forall x \in \Re$.

This TIFN is denoted by $A_{TIFN}=\left(a_1,b_1,c_1;a_1^/,b_1,c_1^/\right)$.

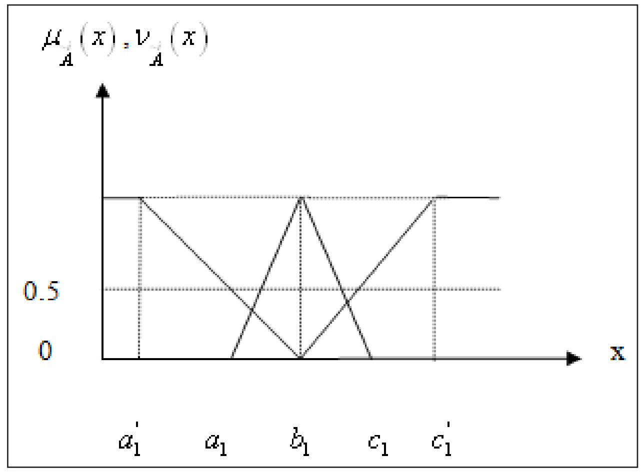

Fig. 2: Membership and non-membership function of TIFN

1. Some arithmetic operations of Intuitionistic Fuzzy Number based on cuts method:

❐ **Properties 3.1**

- If TIFN $\overset{\sim i}{A}=\left(a_1,b_1,c_1;a_1^/,b_1,c_1^/\right)$ and $y=ka(k>0)$, then $\overset{\sim i}{Y}=k\overset{\sim i}{A}$ is a TIFN $\left(ka_1,kb_1,kc_1;ka_1^/,kb_1,kc_1^/\right)$.
- If $y=ka(k<0)$, then $\overset{\sim i}{Y}=k\overset{\sim i}{A}$ is a TIFN $\left(kc_1,kb_1,ka_1;kc_1^/,kb_1,ka_1^/\right)$.

❐ **Properties 3.2**

If, $\overset{\sim i}{A}=\left(a_1,b_1,c_1;a_1^/,b_1,c_1^/\right)$ and $\overset{\sim i}{B}=\left(a_2,b_2,c_2;a_2^/,b_2,c_2^/\right)$ are two TIFN then $\overset{\sim i}{C}=\overset{\sim i}{A}\oplus\overset{\sim i}{B}$ is also TIFN.

$$\tilde{A}^i \oplus \tilde{B}^i = \left(a_1 + a_2, b_1 + b_2, c_1 + c_2; a_1' + a_2', b_1 + b_2, c_1' + c_2'\right)$$

❒ **Properties 3.3**

If $\tilde{A}^i = \left(a_1, b_1, c_1; a_1', b_1, c_1'\right)$ and

$\tilde{B}^i = \left(a_2, b_2, c_2; a_2', b_2, c_2'\right)$ are two TIFN then $\tilde{P}^i = \tilde{A}^i \odot \tilde{B}^i$ is an approximated TIFN.

$$\tilde{A}^i \odot \tilde{B}^i = \left(a_1 a_2, b_1 b_2, c_1 c_2; a_1' a_2', b_1 b_2, c_1' c_2'\right)$$

Construction and solution procedure of a LPP by Triangular intuitionistic Fuzzy Number (TIFN) using simplex algorithm:

Consider the following steps:

1. **Make a change of variables and normalize the sign of the independent terms.**

   A change is made to the variable naming, establishing the following correspondences: x becomes $\tilde{x}_1$ and y becomes $\tilde{x}_2$. As the independent terms of all restrictions are positive no further action is required. Otherwise there would be multiplied by "-1" on both sides of the inequality (noting that this operation also affects the type of restriction).

2. **Normalize restrictions**

The inequalities become equations by adding slack, surplus and artificial variables as the following table:

Inequality type	Variable that appears
≥	- surplus + artificial
=	+ artificial
≤	+ slack

In this case, a slack variable ($\tilde{x}_3$, $\tilde{x}_4$ and $\tilde{x}_5$) is introduced in each of the restrictions of ≤ type, to convert them into equalities, resulting the system of linear equations:

$$(2,4,5;1,4,6)\tilde{x}_1 + (1,3,4;0,3,5)\tilde{x}_2 + (1,1,1;1,1,1)\tilde{x}_3 = 50$$

$$(2,4,5;1,4,6)\tilde{x}_1 + (4,6,7;3,6,8)\tilde{x}_2 + (1,1,1;1,1,1)\tilde{x}_4 = 100$$

$$(2,4,5;1,4,6)\tilde{x}_1 + (3,5,6;2,5,7)\tilde{x}_2 + (1,1,1;1,1,1)\tilde{x}_5 = 90$$

3. **Match the objective function to zero.**

$$Max\ \tilde{z} = (4,6,7;6,8)\tilde{x}_1 + (8,10,11;7,10,12)\tilde{x}_2$$
$$+ (0,0,0;0,0,0)\tilde{x}_3 + (0,0,0;0,0,0)\tilde{x}_4 (0,0,0;0,0,0)\tilde{x}_5$$

4. Write the initial tableau of Simplex method

The initial tableau of Simplex method consists of all the coefficients of the decision variables of the original problem and the slack, surplus and artificial variables added in second step and constraints (in rows). The Cb column contains the coefficients of the variables that are in the base. The first row consists of the objective function coefficients, while the last row contains the objective function value and reduced costs C_j-Z_j. The last row is calculated as follows: $Z_j = \sum C_{b_i} \times X_i$ for i = 1.....m. Although this is the first tableau of the Simplex method and all C_b are null, so the calculation can simplified.

5. Stopping condition

If the objective is to maximize, when in the last row there is no negative value between discounted costs the stop condition is reached. In that case, the algorithm reaches the end as there is no improvement possibility. The Z_j value is the optimal solution of the problem. Another possible scenario is all values are negative or zero in the input variable column of the base. This indicates that the problem is not limited and the solution will always be improved. Otherwise, the following steps are executed iteratively.

Table AU: 1

	C_j	(4,6,7;2,6,8)	(8,10,11;7,10,12)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)
	BV	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$
(0,0,0;0,0,0)	$\tilde{x}_3$	(2,4,5;1,4,6)	(1,3,4;0,3,5)	(1,1,1;1,1,1)	(0,0,0;0,0,0)	(0,0,0;0,0,0)
(0,0,0;0,0,0)	$\tilde{x}_4$	(2,4,5;1,4,6)	(4,6,7;3,6,8)	(0,0,0;0,0,0)	(1,1,1;1,1,1)	(0,0,0;0,0,0)
(0,0,0;0,0,0)	$\tilde{x}_5$	(2,4,5;1,4,6)	(3,5,6;2,5,7)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(1,1,1;1,1,1)
	Zj	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)
	Cj-zj	(4,6,7;2,6,8)	(8,10,11;7,10,12)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)

Table AU: 2

	C_j	(4,6,7;2,6,8)	(8,10,11;7,10,12)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)
	BV	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$
(0,0,0;0,0,0)	$\tilde{x}_3$	(3/2,2,15/7;1,2,9/4)	(0,0,0;0,0,0)	(1,1,1;1,1,1)	-(1/4,1/2,4/7;0,1/2,5/8)	(0,0,0;0,0,0)
(8,10,11;7,10,12)	$\tilde{x}_2$	(1/2,2/3,5/7;1/3,2/3,3/4)	(1,1,1;1,1,1)	(0,0,0;0,0,0)	(1/7,1/5,1/4;1/8,1/5,1/3)	(0,0,0;0,0,0)
(0,0,0;0,0,0)	$\tilde{x}_5$	(1/2,2/3,5/7;1/3,2/3,3/4)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(8/7,2,11/4;7/8,2,4)	(1,1,1;1,1,1)
	Zj	(4,20/3,55/7;7/3,20/3,9)	(8,10,11;7,10,12)	(0,0,0;0,0,0)	(8/7,2,11/4;7/8,2,4)	(0,0,0;0,0,0)
	Cj-zj	-(0,2/3,6/7;1/3,2/3,1)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	- (2,5/3,11/7;7/3,5/3,3/2)	(0,0,0;0,0,0)

6. Choice of the input and output base variables

First, input base variable is determined. For this, column whose value in Z_j row greater than the all positive value is chosen. In this example it would be the variable X_2. If there are two or more equal coefficients satisfying the above condition (case of tie), then choice the basic variable. The column of the input base variable is called pivot column. Once obtained the input base variable, the output base variable is determined.

The decision is based on a simple calculation: divide each independent term between the corresponding value in the pivot column, if both values are strictly positive (greater than zero). The row whose result is minimum score is chosen.

If there is any value less than or equal to zero, this quotient will not be performed. If all values of the pivot column satisfy this condition, the stop condition will be reached and the problem has an unbounded solution. The term of the pivot column which led to the lesser positive quotient in the previous division indicates the row of the slack variable leaving the base. In this example, it is X_4. This row is called pivot row.

If two or more quotients meet the choosing condition (case of tie), other than that basic variable is chosen (wherever possible). The intersection of pivot column and pivot row marks the pivot value.

1. Update table AU

The new coefficients of the tableau are calculated as follows:

In the pivot row each new value is calculated as:

New value = Previous value / Pivot

In the other rows each new value is calculated as:

New value = Previous value - (Previous value in pivot column * New value in pivot row)

So the pivot is normalized (its value becomes 1).

The tableau corresponding to this second iteration is table 2.

8. End of algorithm

It is noted that in the last row, all the coefficients are ≤ 0; so the stop condition is fulfilled.

The solution is optimal as $C_j - Z_j \leq 0$ for all j. Hence the required solution is x1=(0,0,0;0,0,0) and x2=(25/2, 10,100/11;100/7,10,25/3).

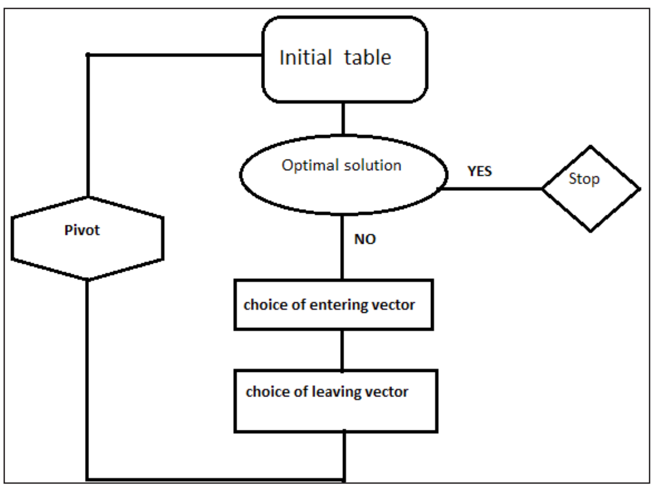

Fig. 1: simplex algorithm

Application

In this paper we are going to solve a linear programming problem by triangular intuitionistic fuzzy number using simplex algorithm. Our problem is described below:

$$Max\ \tilde{z} = (4,6,7;2,6,8)\tilde{x}_1 + (8,10,11;7,10,12)\tilde{x}_2$$

$$(2,4,5;1,4,6)\tilde{x}_1 + (1,3,4;0,3,5)\tilde{x}_2 = 50$$

$$(2,4,5;1,4,6)\tilde{x}_1 + (4,6,7;3,6,8)\tilde{x}_2 = 100$$

$$(2,4,5;1,4,6)\tilde{x}_1 + (3,5,6;2,5,7)\tilde{x}_2 = 90$$

Now this problem rewrite by introducing the slack variables $\tilde{x}_3$, $\tilde{x}_4$ and $\tilde{x}_5$ as,

$$Max\ \tilde{z} = (4,6,7;2,6,8)\ \tilde{x}_1 + (8,10,11;7,10,12)\ \tilde{x}_2 + (0,0,0;0,0,0)\ \tilde{x}_3 + (0,0,0;0,0,0)\ \tilde{x}_4 + (0,0,0;0,0,0)\ \tilde{x}_5$$

Subject to constraint

$$(2,4,5;1,4,6)\tilde{x}_1 + (1,3,4;0,3,5)\tilde{x}_2 + (1,1,1;1,1,1)\tilde{x}_3 = 50$$

$$(2,4,5;1,4,6)\tilde{x}_1 + (4,6,7;3,6,8)\tilde{x}_2 + (1,1,1;1,1,1)\tilde{x}_4 = 100$$

$$(2,4,5;1,4,6)\tilde{x}_1 + (3,5,6;2,5,7)\tilde{x}_2 + (1,1,1;1,1,1)\tilde{x}_5 = 90$$

CONCLUSION

In this paper TIFN and their arithmetic operations are described, we have also solved a simplex problem using TIFN. The procedure of solving simplex problem using TIFN may help us to solve many optimization problems. Our approaches and computational procedures may be efficient and simple to implement for calculation in a Trapezoidal fuzzy environment for all fields of engineering and science where impreciseness occur.

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	C_j	(4,6,7;2,6,8)	(8,10,11;7,10,12)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)
	BV	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$
(0,0,0;0,0,0)	$\tilde{x}_3$	(2,4,5;1,4,6)	(1,3,4;0,3,5)	(1,1,1;1,1,1)	(0,0,0;0,0,0)	(0,0,0;0,0,0)
(0,0,0;0,0,0)	$\tilde{x}_4$	(2,4,5;1,4,6)	(4,6,7;3,6,8)	(0,0,0;0,0,0)	(1,1,1;1,1,1)	(0,0,0;0,0,0)
(0,0,0;0,0,0)	$\tilde{x}_5$	(2,4,5;1,4,6)	(3,5,6;2,5,7)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(1,1,1;1,1,1)
	Zj	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)
	Cj-zj	(4,6,7;2,6,8)	(8,10,11;7,10,12)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)
(0,0,0;0,0,0)	$\tilde{x}_3$	(3/2,2,15/7;1,2,9/4)	(0,0,0;0,0,0)	(1,1,1;1,1,1)	-(1/4,1/2,4/7;0,1/2,5/8)	(0,0,0;0,0,0)
(8,10,11;7,10,12)	$\tilde{x}_2$	(1/2,2/3,5/7;1/3,2/3,3/4)	(1,1,1;1,1,1)	(0,0,0;0,0,0)	(1/7,1/5,1/4;1/8,1/5,1/3)	(0,0,0;0,0,0)
(0,0,0;0,0,0)	$\tilde{x}_5$	(1/2,2/3,5/7;1/3,2/3,3/4)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(8/7,2,11/4;7/8,2,4)	(1,1,1;1,1,1)
	Zj	(4,20/3,55/7;7/3,20/3,9)	(8,10,11;7,10,12)	(0,0,0;0,0,0)	(8/7,2,11/4;7/8,2,4)	(0,0,0;0,0,0)
	Cj-zj	-(0,2/3,6/7;1/3,2/3,1)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	-(2,5/3,11/7;7/3,5/3,3/2)	(0,0,0;0,0,0)

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